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Graphical Regular Logic

Regular logic can be regarded as the *internal language* of regular categories, but the logic itself is generally not given a categorical treatment. In this talk, I'll give an overview of how to understand the syntax and proof rules of regular logic in terms of the free regular category $\mathsf{FRg}(\mathrm{T})$ on a set T. From this point of view, regular theories are certain monoidal 2-functors from a suitable 2-category of contexts—the 2-category of relations in $\mathsf{FRg}(\mathrm{T})$—to that of posets. Such functors assign to each context the set of formulas in that context, ordered by entailment. We refer to such a 2-functor as a *regular calculus* because it naturally gives rise to a graphical string diagram calculus in the spirit of Joyal and Street. The main theorem is that the category of regular categories is essentially reflective in that of regular calculi. Along the way, I'll demonstrate how to use this graphical calculus for regular logic.

References:

[1] B. Fong and D. I. Spivak, Graphical Regular Logic, `https://arxiv.org/abs/1812.05765`.